Simulated Annealing Algorithm for Customer-centric Location Routing Problem

By Eugene Sohn
Advisor: Dr. Mohammad Moshref-Javadi
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Summary:
Traditionally, location routing problem has been solved by minimizing cost. However, the author argues that the location routing problem should be solved by minimizing customer waiting time instead. As e-commerce market continues to grow rapidly and becomes more competitive, consumers are demanding faster deliveries and free shipping. Companies can gain market share in e-commerce and maximize their profits by providing faster deliveries as the consumer culture enters an era of instant gratification. The author introduces mathematical model and three different simulated annealing algorithms to solve capacitated latency location routing problem.

Prior to MIT, Eugene worked in an upstream oil and gas company for three years. He was in Commercial Operations function where he created a capacity model for one of the largest shale gas assets. The capacity model captured the production forecast and existing contractual pipeline capacities to predict future capacity constraints. Eugene received his Bachelor of Business Administration in Supply Chain Management at Texas A&M University, and upon graduating from MIT, Eugene will work in a product planning team at tech company in Cupertino, CA.

KEY INSIGHTS

1. Traditional way of solving location routing problem by minimizing cost is antiquated with recent trends in e-commerce.
2. For larger-sized instances, mathematical model cannot solve the capacitated latency location routing problem (CLLRP) due to the computational inefficiency.
3. Proposed simulated annealing algorithms are effective in solving CLLRP and performs competitively with the algorithms in the literature and mathematical model.

Introduction

In today's world, the e-commerce market is growing rapidly and becoming more competitive. While many players in the industry are attempting to get their share of pie, consumers are demanding faster deliveries and free shipping. This market growth and change in consumer behavior provide an exciting opportunity for companies to compete. In order to meet the new consumer demand, companies need to find better ways to deliver faster, and faster delivery times can be achieved by using an optimization model to plan delivery network and operations. Typically, this optimization model has been based on minimizing cost. However, in the current market, lowest cost is not necessarily the best driver of sales as the consumer culture enters an era of instant gratification. Minimizing customer waiting time will bring better performance and win over market share by providing the quickest delivery service that is expected by the majority of consumers. This paper proposes solving the location routing problem aiming at minimizing customer waiting time with capacitated depots and vehicles.

Methodology

Two approaches are taken to solve Capacitated Latency Location Routing Problem (CLLRP): mathematical model and heuristic algorithm.

Mathematical Model

Because the CLLRP is a combination of two NP-hard problems (Facility Location Problem and Vehicle Routing Problem), it cannot find the optimal solution for real-world size problems in a reasonable time. However, it is important to construct the model and compare with the performance of the heuristic algorithm on small-size problems to guarantee the promising performance of the algorithm.

The assumptions of the mathematical model are the following:

- The number and locations of candidate depot (N_f) are known
- The number of depots to open (N_g) and vehicles (N_v) to use are predetermined
- The capacities of depots (W_g) and vehicles (Q_k) are pre-determined
- All of the demands are satisfied
- The travel time between customer i and j are symmetric

The following notations are used to formulate the problem:

Indices

i,j,u	Represent customers, totally N_c customers
k	Represents vehicle
g	Represents candidate depots, totally N_f

Sets

K	Set of vehicles, $\lvert K \rvert$
G	Set of candidate depots, $\lvert G \rvert = N_f$
V'	Set of customers, $\lvert V' \rvert = N_c$
V	Set of all customers and candidate depots $\lvert V \rvert = N = N_c + N_f$

Parameters

N_v	Number of vehicles
Wg	Capacity of depot g
q_j	Demand quantity at customer *j*
Q_k	Capacity of vehicle *k*
c_{ij}	Travel time between nodes *i* and *j*
N_g	Number of facilities to open
M	Large positive constant

Variables

t_i^k	Arrival time of vehicle *k* at customer *i*
x_{ij}^k	1 if vehicle *k* traverses arc (*i,j*) from customer *i* to customer *j* ; otherwise, 0
f_{gi}	1 if customer *i* is supplied from depot *g*; otherwise, 0
z_g	1, if facility *g* is open; otherwise 0

The mathematical formulation of the CLLRP is the following:

Minimize:

$$\sum_{k\in K, i\in V'} t_i^k \quad (1)$$

s.t.

$$\sum_{i\in V'} f_{gi} q_i \le W_g \ \forall g \in G \quad (2)$$

$$\sum_{j\in V} x_{ij}^k = \sum_{j\in V} x_{ji}^k \ \forall i \in V, \forall k \in K \quad (3)$$

$$\sum_{k\in K, j\in V', i\neq j} x_{ij}^k = 1 \ \forall i \in V' \quad (4)$$

$$\sum_{g\in G} f_{gj} = 1 \ \forall j \in V' \quad (5)$$

$$\sum_{i\in V, j\in V'} x_{ij}^k q_j \le Q_k \ \forall k \in K \quad (6)$$

$$\sum_{g\in G, i\in V'} x_{gi}^k = 1 \ \forall k \in K \quad (7)$$

$$\sum_{u\in V'} x_{gu}^k + \sum_{u\in V\setminus\{i\}} x_{ui}^k \le 1 + f_{gi} \ \forall i \in V', \forall k \in K, \forall g \in G \quad (8)$$

$$t_i^k + c_{ij} - \left(1 - x_{ij}^k\right) M \le t_j^k, \forall i \in V, \forall j \in V', \forall i \neq j, \forall k \in K, \forall g \in G \quad (9)$$

$$\sum_{i\in V'} f_{gi} \le M z_g \ \forall g \in G \quad (10)$$

$$\sum_{i\in V'} f_{gi} \ge z_g \ \forall g \in G \quad (11)$$

$$\sum_{g\in G} z_g = N_g \quad (12)$$

$$t_i^k \ge 0, \forall i \in V, \forall k \in K$$
$$z_g \in \{0,1\} \ \forall g \in G \quad (13)$$
$$x_{ij}^k \in \{0,1\} \ \forall i \in j$$

In this model, the objective function is to minimize the total customer waiting time, which is the sum of the arrival time of the vehicles at customers. Key constraints are depot's capacity (2), vehicle's capacity (6), latency calculation (9), and number of depots to open (12).

Because the Location Routing Problem is a combination of two NP-hard problems (Facility Location Problem and Cumulative Capacitated Vehicle Routing Problem), it is also an NP-hard problem and is only applicable for small-scale problem. Therefore, in the next section we introduce the metaheuristic approach to solve the CLLRP on small and large-scale problems.

Simulated Annealing

Simulated Annealing (SA) is a heuristic algorithm inspired by the annealing process. Annealing is a process in metallurgy where metals are cooled slowly so that the atoms randomly distribute over a longer period of time to increase size of crystals and reduce defects. The atoms move around more quickly when the temperature is high, and they slow down as the temperature cools off. Similar to the atoms in the annealing process, SA accepts a solution more easily when the temperature is high and is stricter when the temperature is low. Therefore, when the temperature is high, SA is able to escape local optimum in order to seek global optimum as the “temperature” of the algorithm decreases.

The SA algorithm begins by creating an initial solution. This solution can be developed randomly or by using a simple algorithm such as nearest neighborhood. SA algorithm then performs operators to the initial solution to develop a new solution and compare with the current best solution to see if an improvement has been made. If the new solution has lower latency than the current best solution, the best solution will be updated with the new solution. If the new solution is not better, then the new solution can still be accepted with a probability determined by Boltzmann function $e^{-\Delta/(kT)}$, where Δ is the latency of new solution – current solution, k is the predetermined constant, and T is the current temperature.

Vehicle 1	Depot 1	C1	C2	C3	C4	-
Vehicle 2	Depot 2	C5	C6	C7	C8	-

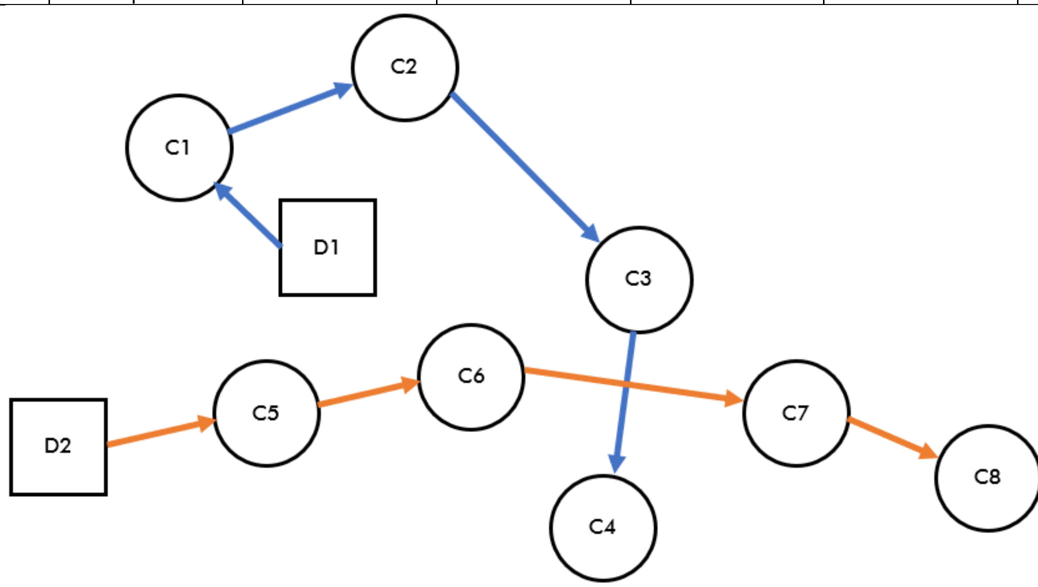

Figure 1: Visualization of the Solution Representation

Figure 1 shows a solution representation after the initial solution has been formulated. Each row is a vehicle’s route that begins from a depot and delivers to customers. For example, in the first row in table in **Error! Reference source not found.**, Vehicle 1 leaves Depot 1 to deliver to Customer 1, Customer 2, Customer 3, and Customer 4. This matrix will be used in Simulated Annealing algorithm to find more optimal solutions.

All three simulated annealing algorithms will use six parameters: T_0, T_f, α, β, over_capacity, and K. T_0 represents the initial temperature, and T_f represents final temperature. The algorithm will begin at T_0 and after each iteration, the temperature will be reduced by T=T* α until T reaches T_f. α represents the rate of cooling. β measures the weight of the overcapacity penalty, while over_capacity measures the total amount of demand that is over capacity in a solution for both vehicles and depots. Finally, K is the Boltzmann constant used in the probability function ($e^{-\Delta/(kT)}$) to determine whether to accept a new solution or not. A well-designed algorithm requires a well-thought-out parameter in order for the model to work effectively, and it is crucial to run the algorithm with different parameters to determine an optimal parameter. After several experiments with different parameters, Table 1 shows the most effective parameter found during the experiment.

Table 1: SA Algorithm Parameters

SA1 Parameters	Values	SA2 Parameters	Values	SA3 Parameters	Values
T_0	25	T_0	25	T_0	25
T_f	0.1	T_f	0.1	T_f	0.1
α	0.99	α	0.99	α	0.99
β	400	β	400	β	400
K	1.0/8.0	K	1.0/8.0	K	1.0/8.0
N-prime_operator	150% of # of Customers	N-operator	75% of # of Customer	N-non_local_operator	# of Customers
N-local_operator	10			N-local_operator	10

Figure 2 shows the visual representation of a specific problem instance's best solution. Green boxes are opened depots. Red box is a depot not used. Each color of the line represents a vehicle and the direction the vehicle is go. For example, Purple line shows that a vehicle is leaving Depot 5 to deliver to Customer 8 and Customer 7.

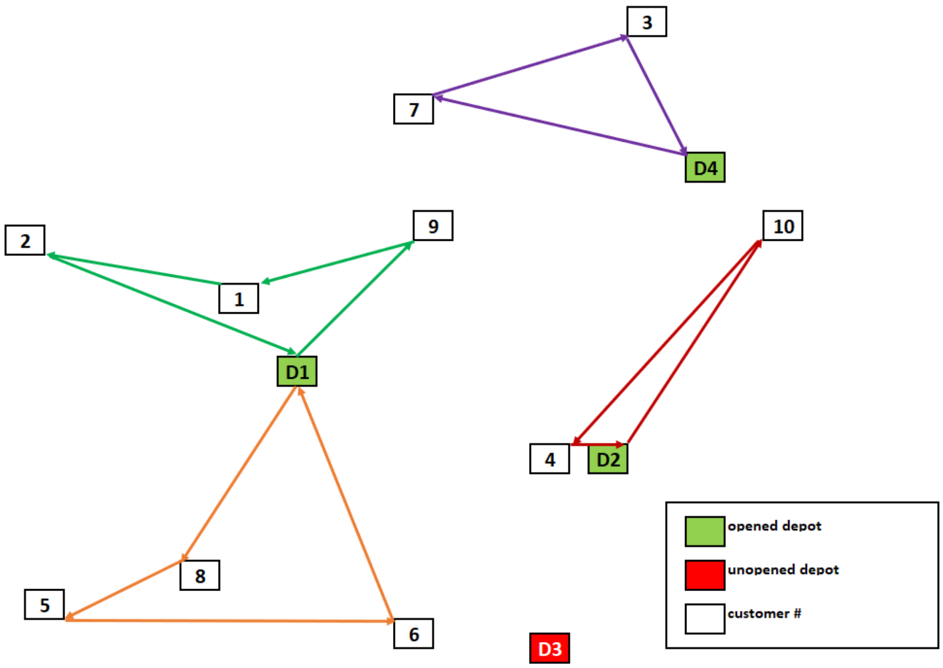

Figure 2: Visual Representation of a Problem Instance's Best Solution

Conclusion

In this paper, we considered a customer-centric location routing problem, so-called the Capacitated Latency Location Routing Problem (CLLRP). Unlike a typical LRP, this paper aims at minimizing the total customer waiting time, instead of minimizing the total costs. In addition to a mathematical model, three different variations of simulated annealing algorithms were proposed based on the strategies to use the operators of the algorithms. SA1 adaptively choses the operators, SA2 sequentially applies the non-local and local operators, and SA3 employs the local operators iteratively after the non-local operators. These algorithms use the nearest neighborhood and probabilistic centrality algorithm to find an initial solution which is then improved using five different operators. The algorithms were compared on Prins et al. (2006) benchmark problem set and the best performing SA was then compared with the mathematical model. Comparing with the LLRP results in Moshref-Javadi and Lee (2016), the proposed SAs show promising results with low gap. Overall, SA2 performed the best in finding a solution with minimum waiting time; however, it also took longer time to compute as the customer size increased. SA3 performed as well as SA2 while having lower computation time.

If companies are looking to revamp their last mile delivery to adjust to new consumer behavior, they should look into minimizing total delivery time instead of total cost.