

# Aggregate Production Planning for Engineer-to-Order Products

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**Summary:** The contract manufacturing industry is growing and shifting from standard to customized engineer-to-order products. ETO orders have production process uncertainties, which increase costs. We formulate an Aggregate Production Planning (APP) model as a deterministic, multi-product, multi-stage, and multi-period linear programming (LP) model. The model minimizes the total production cost by balancing the in-house production, inventory holding, outsourcing, overtime, and backlogging costs. We analyze multiple scenarios to estimate the impact of uncertainties on the total product cost.



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## KEY INSIGHTS

1. Production process uncertainty of ETO orders decreases a company's profitability due to additional manufacturing expenses and late delivery penalties
2. APP LP optimization model reduces the total production cost by planning in advance outsource, hiring, overtime hours, and required inventory holding
3. Cost breakdown and shadow price analysis help a company make better decisions on resource use and allocation

## Introduction

The contract manufacturing industry is growing and shifting towards highly customized engineer-to-order (ETO) products. Different from standard products, ETO orders have more production process uncertainties because their design specifications and therefore the production process can be changed after the orders have been accepted. Design changes expose the firm to potential increases in production costs, the risk of late delivery and associated penalties, negatively impacting the firm's bottom line.

Our sponsor company, a contract manufacturer, faces the problem of increasing production cost for their ETO orders. We found that the aggregate planning and cost estimation processes are isolated and neither process considers potential production disruptions.

The goal of the project is to develop a planning tool and show its potential applications. Our project first develops a deterministic linear programming model to optimize the firm's ETO production on an aggregate level. This model provides a solution with the minimum total production cost. We run the model with different production scenarios to develop recommendations for dealing with potential production disruptions. Sensitivity analysis is conducted to gain insights into manufacturing resources investments.

This project will benefit contract manufacturers by providing an APP solution for cost minimization and preparation for possible production disruptions of the ETO orders. We believe that its benefits will go beyond the contract manufacturing industry to other sectors with customized orders.

## Methodology

We develop a Linear Programming (LP) model as a fundamental framework for the APP. This model optimizes a multi-product, multi-stage and multi-time period production process (process map shown in *Figure 1*) to yield an aggregate production plan with the minimum total cost by balancing the internal production, inventory holding, outsourcing, and backlogged orders.

While this APP model gives the optimal production plan for deterministic demand and process, ETO orders have inherent process uncertainties. To estimate the impact of possible process disruptions, we compare a base scenario with alternative scenarios assuming they have a uniform probability distribution.

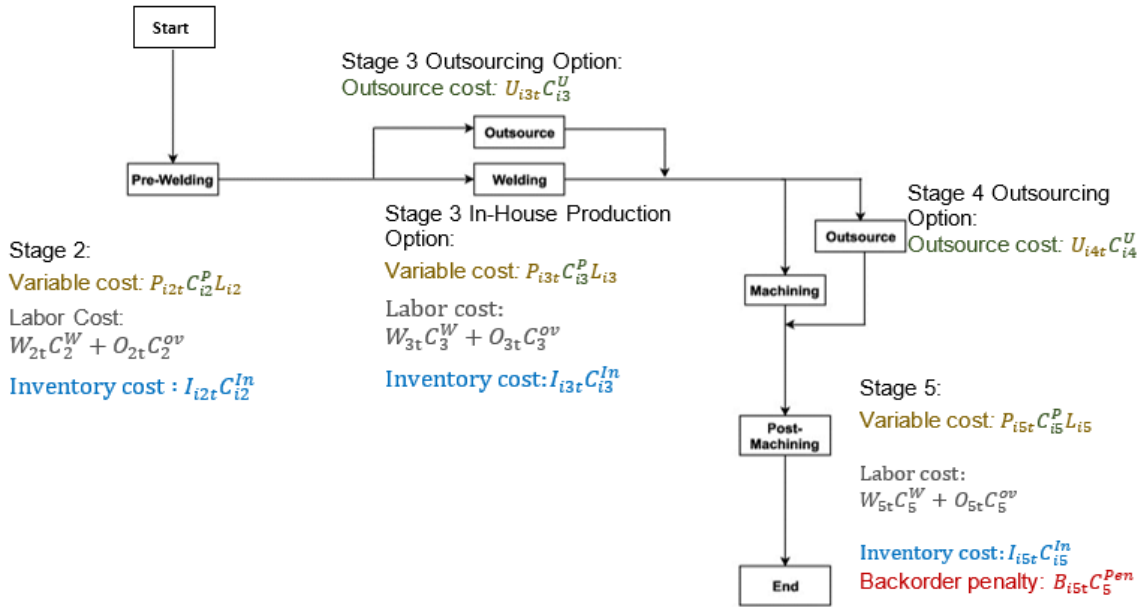


Figure 1: Process Model for the LP Optimization Model

We compare the total production cost for the three model options:

**Option 1:** Hiring, overtime hours, and outsourcing is allowed. It represents the company's current operations.

**Option 2:** Outsourcing and overtime are not allowed, but hiring is allowed. It could be used for

the regular internal manufacturing planning because overtime and outsourcing are generally more expensive.

**Option 3:** No hiring allowed, but overtime hours and outsourcing is allowed. It represents an urgent reaction as hiring is impossible on short notice.

$$\text{Min } z = \sum_i \sum_s \sum_t (P_{ist} L_{is} C_{is}^P I_{ist} + I_{ist} C_{is}^{In} + U_{ist} C_{is}^U) + \sum_s \sum_t (W_{st} C_s^W + O_{st} C_s^{Ov}) + \sum_i \sum_t (B_{it} C_i^{Pen}) \quad \forall i, \forall s, \forall t \in \{1 \dots T\} \quad (1)$$

s.t.:

$$\sum_i (P_{ist} L_{is} - (HW_{st} + O_{st})) * z \leq 0 \quad \forall s, \forall t \in \{1 \dots T\} \quad (2)$$

$$\sum_i (P_{ist} L_{is} - (P_s^{Max} * 24 * 6)) * z \leq 0 \quad \forall s, \forall t \in \{1 \dots T\} \quad (3)$$

$$W_{st} - W_{s,t-1} - A_{st} = 0 \quad \forall s, \forall t \in \{1 \dots T\} \quad (4)$$

$$O_{st} - O^{max} W_{st} * z \leq 0 \quad \forall s, \forall t \in \{1 \dots T\} \quad (5)$$

$$U_{ist} - U_{is}^{Max} \leq 0 \quad \forall i, \forall s, \forall t \quad (6)$$

$$I_{is,t-1} - I_{ist} + P_{ist} + U_{ist} - (P_{is+1,t} + U_{is+1,t}) = 0 \quad \forall i, \forall s \in \{1 \dots S-1\}, \forall t \in \{1 \dots T\} \quad (7)$$

$$I_{is,t-1} - I_{ist} + P_{ist} + U_{ist} - D_{it} - B_{it-1} + B_{it} = 0 \quad \forall i, s=S, \forall t \in \{1 \dots T\} \quad (8)$$

$$A_{st} W_{st} I_{ist} P_{ist} U_{ist} O_{st} B_{it} \geq 0 \quad \forall s, \forall t \quad (9)$$

#### Indices:

$i$  – product,  $1 < i < N$

$s$  – production stage,  $1 < s < S$

$t$  – time period,  $0 < t < T$

#### Decision variables:

$A_{st}$  – employees to hire at start of period  $t$ , stage  $s$

$W_{st}$  – employees in the end of period  $t$ , stage  $s$

$O_{st}$  – overtime hours to work in period  $t$ , stage  $s$

$I_{ist}$  – units of inventory, product  $i$ , end of period  $t$ , stage  $s$

$P_{ist}$  – units to produce internally, period  $t$ , stage  $s$ , product  $i$

$U_{ist}$  – units to outsource, period  $t$ , stage  $s$ , product  $i$

$B_{it}$  – units of backlog by product by week

#### Input data:

$D_{it}$  – demand for product  $i$  period  $t$ , units

$L_{is}$  – production time for stage  $s$  product  $i$ , hours/unit

$W_{s0}$  – workforce at week 0 stage  $s$ , # of employees

$H$  – working hours, hours/person/week

$O^{max}$  – max hours of overtime, hours/person/week

$U_{is}^{max}$  – max outsourcing product  $i$  stage  $s$ , units/week

$P_s^{Max}$  – production equipment by stage, units

$I_{is0}$  – inventory at week 0 product  $i$  stage  $s$ , units

$C_{is}^P$  – production cost for product  $i$  stage  $s$ , ¥/hour

$C_s^{Ov}$  – cost of overtime hour for stage  $s$ , ¥/hour

$C_s^W$  – cost of employee for stage  $s$ , ¥/person/week

$C_{is}^{In}$  – inventory holding cost product  $i$  stage  $s$ , ¥/unit/week

$C_{is}^U$  – outsource cost product  $i$  stage  $s$ , ¥/unit

$C_i^{Pen}$  – late delivery penalty product  $i$ , ¥/unit/week

$z$  – in-house capacity used, % (0-100%)

Comparing the three options under all scenarios provides a direction for cost reduction and optimal plan calculation.

We perform the following analysis:

- Compare the minimum of Option 1 and Option 2 with Option 3 total expected costs to estimate the benefit for the company of using APP in advance.
- Use shadow price analysis to identify excessive and insufficient capacities (equipment and workforce).
- Make a cost breakdown analysis to identify cost drivers for different model options and scenarios.
- Calculate buffer capacity for the base scenario by matching its cost with the total expected cost for all scenarios. This buffer capacity for the base scenario keeps the total cost at the expected level and allows to prepare for possible uncertainties.

### Results

We use a dataset from the sponsor company for a 20-week planning horizon with the demand for 20 ETO products.

Cost breakdown for the base scenario is shown in *Figure 2*. Employees' salaries and outsourcing are the top cost drivers. Results of the total expected production costs are shown in *Figure 3*. Contrary to our expectation, Option 1 is cheaper than Option 2, as new employees hired at the beginning of the planning period is expensive and employees cannot be fired later even if demand is low. For the extreme scenarios, the demand cannot be met due to equipment capacity constraints, resulting in backlogged orders and penalties. A detailed analysis shows that for some products and stages outsourcing is cheaper than internal production. However, with other demand and production time inputs, it is possible that Option 2 can have the lower cost.

For the analyzed data, APP with overtime, hiring, and outsourcing (Option 1) reduces the production cost of the company by 11.86% compared to Option 3 (when no planning is performed in advance).

Shadow price analysis shows that the firm has too many employees in stage 3 and not enough in stage 5. One employee removed from Stage 3 and one employee added to Stage 5 reduces the total expected cost for Option 1 model by 0.52% while keeping total headcount unchanged. A similar analysis for equipment does not show significant cost reduction while significant investments would be required confirming that machine capacity is not a severe bottleneck.

One important result of the planning tool is a detailed production plan for each product which includes volume in units produced in-house and outsourced, inventories kept, employees hired, overtime hours, and expected backlog.

To identify the buffer capacity corresponding to cost of potential planning interruptions, we use the base scenario and include buffer capacity with 1% increments to the model calculation until total production cost matches total expected cost for all the scenarios. For our data set recommended buffer capacity, which equals the base scenario cost and the total expected cost, is 7%.

With the 7% buffer capacity, re-allocation of the workforce, model use with hiring, overtime and outsourcing, total cost reduction for the company is 12.32%.

### Conclusions

This project formulated an APP LP model for the sponsor company solved it for multiple scenarios under three model options and analyzed cost breakdown. It recommended a methodology for setting buffer capacities to prepare for process uncertainties, as well as suggested changes in

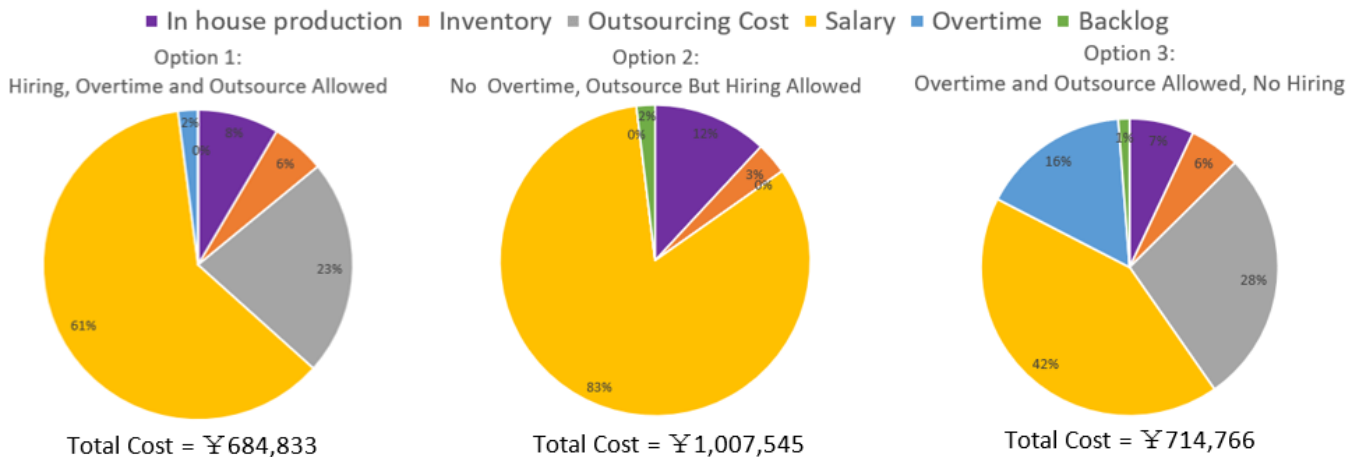


Figure 2: Cost Components Breakdown for the Base Scenario under the Three Model Options

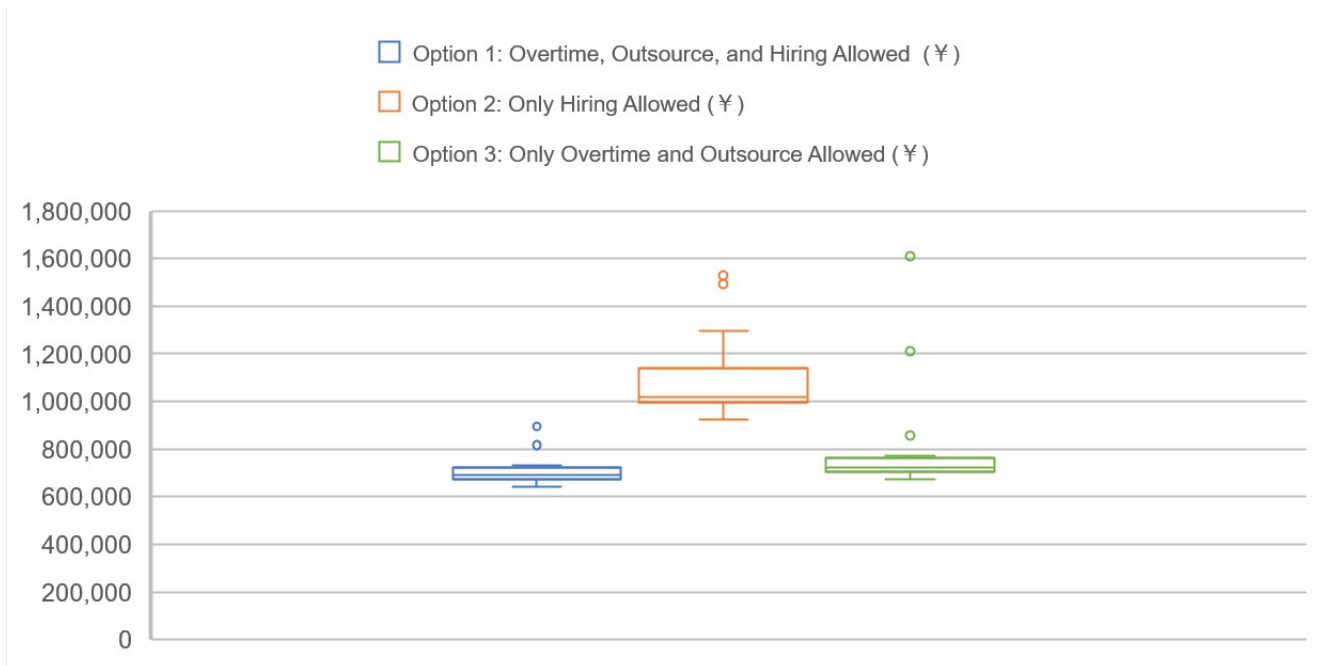


Figure 3: Box Plot of the Total Expected Costs (for All Scenarios) under the Three Model Options

available capacities based on a shadow price analysis.

We recommend the following approach to the APP process for the company:

1. Continue using a combination of in-house production, overtime hours and outsourcing;
2. Calculate buffer capacity based on a scenarios analysis to prepare for production time uncertainty;
3. Use shadow price analysis for better resource allocation and capacity investments;

4. Conduct regular cost analysis towards a more strategic outsourcing practice to outsource orders which are cheaper to produce using sub-contractors than in-house;

Future research may advance the model towards explicitly modeling a decision to set buffer capacity and process uncertainty

We believe that the developed approach and the formulated model can be expanded as a general methodology for any industry and company involved in manufacturing of ETO products.